

Holography of a Composite Inflaton

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We study the time evolution of a brane construction that is holographically dual to a strongly coupled gauge theory that dynamically breaks a global symmetry through the generation of an effective composite Higgs vev. The D3/D7 system with a background magnetic field or non-trivial gauge coupling (dilaton) profile displays the symmetry breaking. We study motion of the D7 brane in the background of the D3 branes. For small field inflation in the field theory the effective Higgs vev rolls from zero to the true vacuum value. We study what phenomenological dilaton profile generates the slow rolling needed, hence learning how the strongly coupled gauge theory's coupling must run. We note that evolution of our configuration in the holographic direction, representing the physics of the strong interactions, can provide additional slowing of the roll time. Inflation seems to be favoured if the coupling changes by only a small amount or very gently. We speculate on how such a scenario could be realized away from $N=4$ gauge theory, for example, in a walking gauge theory.

I. INTRODUCTION

In recent years the AdS/CFT Correspondence [1–4] has emerged as a very powerful tool for studying strong coupling problems. It provides a weakly coupled gravitational/string description of strongly coupled gauge dynamics allowing computation of many aspects of the dynamics. Originally the duality was proposed for the $N = 4$ super Yang Mills theory but it has been widely expanded to other theories over the past ten years. Amongst those theories for which gravity duals exist are several examples of strongly coupled gauge theories with fundamental quark fields and which break chiral symmetries by the formation of a quark condensate as QCD does [5–8]. In this paper we wish to use these tools to make a first study of time dependence in such symmetry breaking theories. We expect strongly coupled gauge theories to undergo phase transitions between a high temperature chirally symmetric phase and a low temperature chiral symmetry breaking phase. Holography should for the first time allow us to analyse the evolution of such theories between these different phases. The simplest example of where such a transition might play an important role in the evolution of our Universe is in inflation and in this paper we will concentrate on that possibility.

Inflation (see for example [9]) is now a key part of the standard cosmological model of the Universe, supported by many pieces of astrophysical data. The usual description involves one or more scalar fields slow-rolling from an unstable point in a potential to the true vacuum. The origin for such a scalar field remains unclear though. Indeed, until evidence for supersymmetry is found in nature, fundamental scalar fields are formally unnatural in field theory as a result of the hierarchy problem (radiative corrections cause their mass to naturally grow to the Planck scale). This need be no obstacle to the paradigm

though since many strongly coupled systems have scalar order parameters describing their dynamics that could play the same role. For example, in QCD the gauge interactions dynamically generate a quark bilinear condensate ($\langle \bar{q}q \rangle$) that breaks chiral symmetries - one may think of that bilinear as the expectation value of a composite scalar field. The main reason to work with fundamental scalars in cosmology is simply that we have had no tools to study strong coupling problems such as the condensation in QCD. The AdS/CFT Correspondence for the first time allows us to analyse just such a scenario though.

We will concentrate on a particular duality which we believe to be the simplest example of holography with fundamental quark fields [10–14]. We wish to stress that we do not consider the specific degrees of freedom of the theory too crucial - it is some strongly coupled gauge theory that generates quark condensates. We hope, in the spirit of AdS/QCD models [15, 16], that it reflects broad aspects of many strongly coupled systems. The specific gauge theory is constructed from the D3/D7 system¹ in type IIB string theory which we will describe in detail below. The theory is the large N_c $N = 4$ $U(N_c)$ gauge theory with a small number of quark hypermultiplets. We will work in the quenched approximation [12] (appropriate when $N_f \ll N_c$) which on the gravity dual side corresponds to treating the D7 branes as probes in the AdS metric generated by the D3 branes. There is a $U(1)$ symmetry (a remnant of the $SU(4)$ R-symmetry of the $N = 4$ theory) which is broken when a quark condensate forms [6]. Several mechanisms for triggering this condensation have been explored. The cleanest is when

¹ The D3/D7 system has been used to construct an inflation model in [17] and subsequent literature. The motivation in those models is rather different since they are not holographic descriptions of a strongly coupled gauge theory but instead assumed to describe weakly coupled fields with the extra dimensions compactified. The dilaton profiles and brane motions we will consider are very different being inspired by the gauge duality and the lessons we will seek to extract are for strongly coupled gauge dynamics.

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a background magnetic field is introduced [18, 19]. Running of the coupling also causes quark condensation as has been shown in back-reacted dilaton flow geometries [6, 7] and models with a phenomenologically imposed dilaton profile [8]. The quark condensate can be determined in these models and an effective IR quark mass is generated. The theories display a massless pion-like Goldstone field and a massive sigma field (since we are at large N_c it is stable) that is the effective Higgs particle.

We will seek to learn about a strongly coupled counterpart to a “new inflation” or “small field inflation” model [20]. One assumes a high temperature phase where the effective Higgs scalar’s vev is zero and a low temperature phase where the vev is non-zero. A second order phase transition should connect these phases so that the vacuum is left at the unstable zero vev point of the low temperature theory and then rolls to the true vacuum. For inflation the potential needs to be very flat near the origin so the roll takes a long time. Of course, other possible inflationary scenarios exist with multiple scalar fields or where the scalar expectation value is initially large relative to the vacuum value and it would be interesting to investigate these ideas holographically too in the future.

In principle one could imagine taking a holographic model and tracking it through the finite temperature transition. On the gravity side temperature is introduced through a black hole in the AdS geometry. The D3/D7 model we investigate has been shown to display both first order (eg with a magnetic field [18, 19]) and second order (eg with a magnetic field and chemical potential [19]) symmetry restoration transitions. Ideally one would work in a time dependent background describing a shrinking black hole. In fact such geometries are known [21] and a first study of D7 branes moving slowly near a potential minimum in those geometries can be found in [22]. Dealing with such branes when they touch the black hole is hard though - we hope to study this problem in a future publication.

Our goal in this paper is more limited though and similar to the usual simplest field theory analysis of inflation models. We will assume that below the phase transition the theory is well described by the $T = 0$ theory. We will invoke initial conditions that place the vacuum in the symmetric phase and then watch it roll to the true vacuum that breaks the symmetry. These initial conditions assume the existence of a second order transition.² Our first results are numerical simulations of this roll in the theory where a magnetic field is inducing the symmetry breaking. As we said above this theory actually has a first order thermal transition rather than a second order one - the scenario serves to demonstrate the formalism

though. The results would be relevant to the phase transition within a highly supercooled bubble. Here there is no fine tuning to make the potential particularly flat so the model would make a poor inflation model. It serves as a proof of the numerical techniques we use to study the problem and it is impressive that a relatively simple computation can track such a transition in a strongly coupled gauge theory.

One does not imagine that a magnetic field induced symmetry breaking is likely in a realistic model of inflation. We therefore turn to a more phenomenological model in which we embed the D7 brane in an AdS geometry but with a hand chosen and non-back reacted dilaton profile. Whilst this is not a completely kosher string dual we hope that it allows us to understand broadly the dependence of a strongly coupled theory’s dependence on its running coupling. We used a similar approximation in [8] to study walking gauge dynamics and found agreement between the gravity dual description and the usual expectations for the coupling dependence of the chiral symmetry breaking. The model is sufficiently simple that we can understand how the running of the gauge coupling affects the potential for the effective Higgs mode. We attempt to engineer models with a slow roll between the symmetric and broken phases. We also show that part of our success in slowing the roll time is due to additional dynamics in the holographic radial coordinate of the dual description. This dynamics reflects the strong coupling dynamics of the gauge theory which seems under some circumstances to favour inflation. From this analysis we conclude that gauge theories displaying a small increase in the gauge coupling, preferably over a wide energy regime, are liable to make good inflation models.

Finally we briefly speculate on how IR conformal fixed points may well be common in strongly coupled gauge dynamics and suggest a wider set of theories that might display running of the coupling like that we find gives inflation. Obvious examples are walking gauge theories.

II. INFLATION

Let us very briefly review the standard inflation model [9]. Based on the cosmological principle we assume the Friedmann-Robertson-Walker metric of the form

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right), \quad (1)$$

and the matter energy momentum tensor of a perfect fluid $T^\mu_\nu = \text{diag}(-\varepsilon, p, p, p)$. For simplicity we will work in the $k = 0$ case. Then the general Einstein equations describing the expansion of the Universe are reduced to two equations:

$$H^2 = \frac{1}{3}\varepsilon \quad : \text{Friedmann equation}, \quad (2)$$

$$\dot{\varepsilon} + 3H(\varepsilon + p) = 0 \quad : \text{Fluid equation}, \quad (3)$$

² Also see [23] for a time-dependent D5 brane embedding dynamics in the context of a quantum quench. Non-equilibrium dynamics has been studied also in holographic superconductor [24]

where $H(t) \equiv \dot{a}/a$ is the Hubble parameter and the scale factor $a(t)$ is readily determined from $H(t)$ as

$$a(t) = a(0) \exp \left(\int_0^t H(t') dt' \right). \quad (4)$$

Equations (2) and (3) can be combined to form a so called “acceleration equation”

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\varepsilon + 3p), \quad (5)$$

from which the condition for inflation can be expressed in terms of the energy momentum tensor.

$$\text{Inflation} \quad \Leftrightarrow \quad \ddot{a} > 0 \quad \Leftrightarrow \quad p < -\frac{\varepsilon}{3}. \quad (6)$$

Thus a natural question for inflation is asking what kind of matter and dynamics can generate sufficient negative pressure (6).

For example let us consider a scalar field.

$$S = \int d^4x \sqrt{-g} \frac{1}{2} \mathcal{R} + S_M, \quad (7)$$

where \mathcal{R} is Ricci scalar and

$$S_M = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right], \quad (8)$$

in unit of $8\pi G = 8\pi m_P^{-2} = 1$. m_P is the Plank mass. From a scalar matter action the energy-momentum tensor of a homogeneous field $\varphi(t)$ can be obtained as

$$\varepsilon = -T_0^0 = \frac{1}{2} \dot{\varphi}^2 + V(\varphi), \quad p = T_i^i = \frac{1}{2} \dot{\varphi}^2 - V(\varphi). \quad (9)$$

The inflation condition (6) is rephrased as

$$p < -\frac{\varepsilon}{3} \quad \Leftrightarrow \quad \dot{\varphi}^2 < V(\varphi). \quad (10)$$

Whenever the kinetic term is small compared to the potential energy there will be inflation. Then by analogy to mechanics we may say φ is *rolling slowly*.

Let us apply this slow roll condition to the equations of motion (2) and (3).

$$H^2 = \frac{1}{3} \varepsilon = \frac{1}{3} \left(\frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right), \quad (11)$$

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0, \quad (12)$$

where the fluid equation (12) is nothing but the equation of motion for φ , which can be obtained from (8).

If the scalar field stays in some part of the potential with $\dot{\varphi}^2 \ll V$ then $H \sim V(\varphi(t))/3$ from (11). Furthermore if $\ddot{\varphi}$ is sufficiently smaller than the other terms in (12) then $V(\varphi)$ can be considered as constant, say V_0 , for

a long time. Thus the scale factor (4) in the slow rolling range reads

$$a(t) \sim a(0) e^{Ht} = a(0) e^{\sqrt{\frac{8\pi V_0}{3}} \frac{1}{m_P} t}, \quad (13)$$

where we have reinserted the Planck mass. This is the de Sitter limit zeroth order approximation of an inflating Universe, which we will adopt in this paper. To have a graceful exit from inflation the Hubble parameter must be time dependent and fall to zero at late times. In an inflating regime the space is equivalent to de Sitter space with a cosmological constant $\Lambda = 8\pi m_P^{-2} V_0$, where V_0 is interpreted as the vacuum energy density. The amount of inflation is specified by the number of e-folds given by

$$N_e \equiv \log \frac{a(t_e)}{a(0)} = H t_e = \sqrt{\frac{8\pi V_0}{3}} \frac{t_e}{m_P}, \quad (14)$$

where t_e is the time when inflation ends (when slow roll conditions are violated) starting from $t = 0$. One needs N_e to be more than 60 phenomenologically.

Note that a larger H yields a more inflationary evolution. H is nothing but a friction coefficient when we interpret (12) as an equation for the classical particle trajectory ($\varphi(t)$) under the potential V . Thus it is natural that a larger friction induces a slower rolling of the particle.

In this paper we will replace the scalar field sector with a strongly coupled gauge theory. The gauge dynamics will generate a wine bottle shaped potential with a non-zero condensate of a quark bilinear at the minimum. We will phenomenologically adjust the running of the gauge theory’s coupling constant to control the shape of the potential. We hope to learn how a gauge theory’s coupling must run to generate inflation. Clearly our task is to compute the stress energy tensor (ε and p) of a strongly coupled gauge theory so we can substitute them into the Friedmann and fluid equations above. We must find the equivalent of the Euler Lagrange equation for the scalar field φ in the inflating background for the time evolution of the strongly coupled gauge dynamics. To do this we will turn to Gauge/Gravity duality.

III. HOLOGRAPHIC DESCRIPTION OF A STRONGLY COUPLED GAUGE THEORY

A. Brane Construction

First let us review the gravity dual description of the symmetry breaking behaviour of our strongly coupled gauge theory[5–8]. To begin with we will work in a time independent flat space (*i.e.* there is no inflation here and $a(t) = 0$).

Dp-branes are p dimensional membrane-like objects on which the ends of open strings can be fixed. The weak coupling picture for our D3/D7 set up is shown in Fig 1 - there are N D3 branes and the lightest string states with

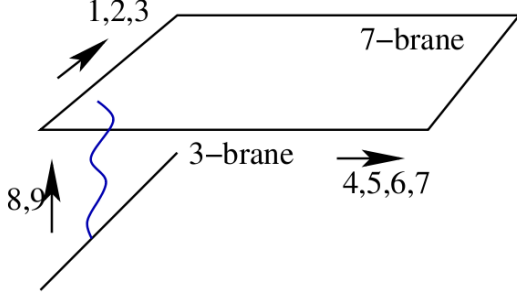


FIG. 1: A sketch of the D3/D7 construction that our model is based on. 3-3 strings generate the gauge theory whilst 3-7 strings are quark fields.

both ends on that stack generate the adjoint representation fields of the $N = 4$ gauge theory. Strings stretched between the D3 and the D7 are the quark fields lying in the fundamental representation of the $SU(N)$ group (they have just one end on the D3).

In the strong coupling limit the D3 branes in this picture are replaced by the geometry that they induce [1–4]. We will consider a gauge theory with a holographic dual described by the Einstein frame geometry $AdS_5 \times S^5$

$$ds^2 = \frac{1}{g_{UV}} \left[\frac{r^2}{R^2} dx_4^2 + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dw_5^2 + dw_6^2) \right], \quad (15)$$

where we have split the coordinates into the $x_4(x_{3+1})$ of the gauge theory, the ρ and Ω_3 which will be on the D7 brane world-volume and two directions transverse to the D7, w_5, w_6 . The radial coordinate, $r^2 = \rho^2 + w_5^2 + w_6^2$, corresponds to the energy scale of the gauge theory. The radius of curvature is given by $R^4 = 4\pi g_{UV}^2 N_c \alpha'^2$ with N_c the number of colours. In addition we will allow an arbitrary dilaton as $r \rightarrow 0$ to represent the running of the gauge theory coupling

$$e^\Phi = g_{YM}^2(r^2) = g_{UV}^2 \beta(r^2), \quad (16)$$

where the function $\beta(r) \rightarrow 1$ as $r \rightarrow \infty$. The $r \rightarrow \infty$ limit of this theory is dual to the $N = 4$ super Yang-Mills theory and g_{UV}^2 is the constant large r asymptotic value of the gauge coupling.

We will introduce a single D7 brane probe [12] into the geometry to include quarks - by treating the D7 as a probe we are working in a quenched approximation although we can reintroduce some aspects of quark loops through the running coupling's form if we wish (or know how). This system has a $U(1)$ symmetry acting on the quarks, corresponding to rotations in the w_5 - w_6 plane, which will be broken by the formation of a quark condensate.

In the true vacuum at zero temperature ($T = 0$) the brane will be static. We must find the D7 embedding function *e.g.* $w_5(\rho) \equiv L(\rho)$ with $w_6 = 0$. The DBI action

in Einstein frame is given by

$$\begin{aligned} S_{D7} &= -T_7 \int d^8 \xi e^\Phi \sqrt{-\det P[G]_{ab}} \\ &= -\tilde{T}_7 \int d^4 x d\rho \rho^3 \beta \sqrt{1 + L'^2}, \end{aligned} \quad (17)$$

where $P[G]_{ab}$ is the pull back of the background metric onto the D7 and $L' \equiv \partial_\rho L(\rho)$. $T_7 = 1/(2\pi)^7 \alpha'^4$ and $\tilde{T}_7 = 2\pi^2 T_7 / g_{UV}^2$ when we have integrated over the 3-sphere on the D7. The equation of motion for the embedding function is therefore

$$\partial_\rho \left[\frac{\beta \rho^3 L'}{\sqrt{1 + L'^2}} \right] - 2L \rho^3 \sqrt{1 + L'^2} \frac{\partial \beta(r^2)}{\partial r^2} = 0. \quad (18)$$

The UV asymptotic of this equation, provided the dilaton returns to a constant so the UV dual is the $N = 4$ super Yang-Mills theory, has solutions of the form

$$L = m + \frac{c}{\rho^2} + \dots, \quad (19)$$

where we can interpret m as the quark mass ($m_q = m/2\pi\alpha'$) and c is proportional to the quark condensate.

The embedding equation (18) clearly has regular solutions $L = m$ when g_{YM}^2 is independent of r - the flat embeddings of the $N = 2$ theory [12, 13]. Equally clearly if $\partial \beta(r^2)/\partial r^2$ is non trivial in L then the second term in (18) will not vanish for a flat embedding.

There is always a solution $L = 0$ which corresponds to a massless quark with zero quark condensate ($c = 0$). In the high T phase this is the true vacuum. In the symmetry breaking low T geometry this configuration is a local maximum in the potential.

Note that in the particular case when

$$\beta = \sqrt{1 + \frac{B^2}{(\rho^2 + L^2)^2}}, \quad (20)$$

the DBI action for the D7 brane is that of the D3/D7 system with a background magnetic field B . This model has been extensively studied in [18, 19]. The action manifestly grows as one approaches $L = \rho = 0$ so the D7 brane is repelled from that point. In Fig 2 we plot the D7 embedding in the magnetic field case.

An interesting phenomenological case is to consider a gauge coupling running with a step of the form

$$\beta = A + 1 - A \tanh[\Gamma(r - \lambda)]. \quad (21)$$

Of course in this case the geometry is not back reacted to the dilaton and the model is a phenomenological one in the spirit of AdS/QCD. This form introduces conformal symmetry breaking at the scale $\Lambda = \lambda/2\pi\alpha'$ which triggers chiral symmetry breaking. The parameter A determines the increase in the coupling across the step. If the coupling is larger near the origin then again the D7 brane will be repelled from the origin. The parameter Γ spreads the increase in the coupling over a region in r of

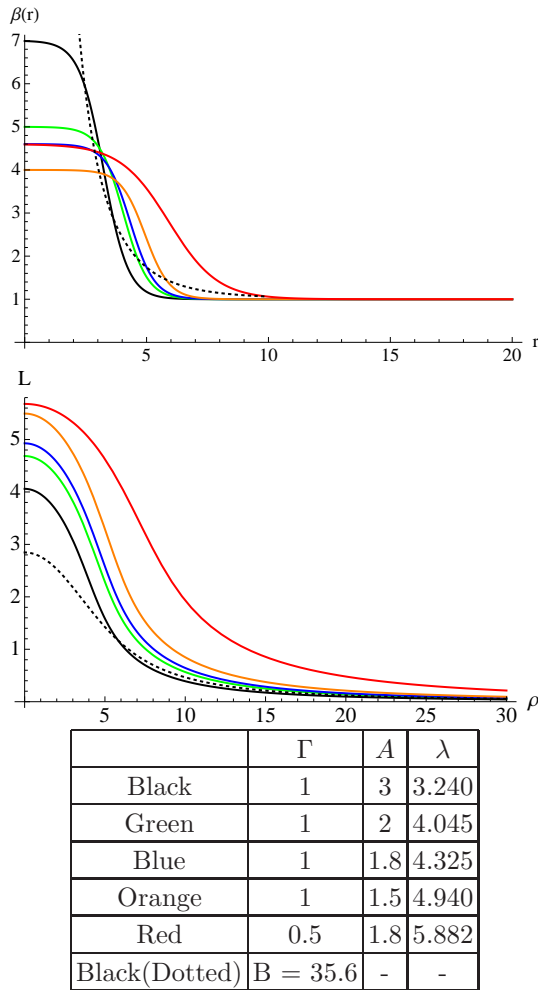


FIG. 2: The coupling flow (16) (top) and the D7 brane embeddings/quark self energy (bottom).

order Γ^{-1} in size - the effect of widening the step is to enhance the large ρ tail of the D7 embedding.

We again display the embeddings for some particular cases in Fig 2. Note that we have chosen parameters here that make the potential difference between the symmetric and symmetry broken phases the same in each case. This is crucial to ensure that we are comparing models that will generate the same cosmological constant and hence the same rate of inflation when the quark condensate is zero. The vacuum energy is given by the DBI action evaluated on the solution. In fact this energy is formally divergent corresponding to the usual cosmological constant problem in field theory. As usual we subtract the UV component of the energy - we do this by subtracting the energy of the symmetry breaking, lowest energy embedding in each case.

The symmetry breaking of these solutions is manifest. The $U(1)$ symmetry corresponds to rotations of the solution in the w_5 - w_6 plane. An embedding along the axis corresponds to a massless quark with the symmetry unbroken (this is the configuration that is preferred at high

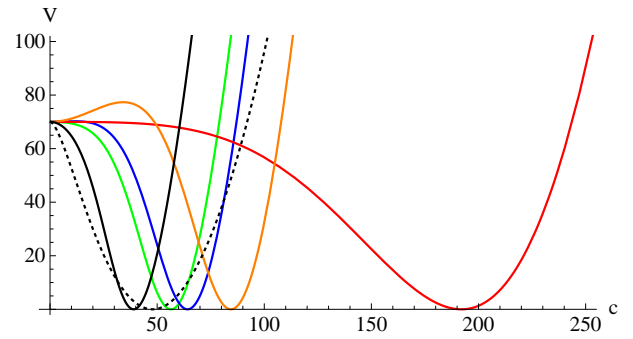


FIG. 3: The approximate potential, $V \equiv S_{D7}/\tilde{T}_7$, for the quark condensate c , from the holographic model for the running couplings and D7 embeddings of Fig 2.

temperature and it has zero condensate c). The vacuum, curved configurations though map onto that case at large ρ (the UV of the theory) but bend off axis breaking the symmetry in the IR.

One can interpret the D7 embedding function as the dynamical self energy of the quark, similar to that emerging from a gap equation [25]. The separation of the D7 from the ρ axis is the mass at some particular energy scale given by ρ - in the $N = 2$ theory where the embedding is flat the mass is not renormalized, whilst with the magnetic field or running coupling an IR mass forms.

B. Approximate Potentials

It is natural to want to plot the potential for the quark condensate using the holographic description. However, this is somewhat ambiguous. The embedding equation determines the D7 embeddings that correspond to the turning points of such a potential. In between these points one would need to find the minimum action configuration that falls off in the UV as c/ρ^2 for arbitrary c and has $L' = 0$ in the IR. This can't be done by a simple numerical shooting because such configurations are not solutions of the Euler Lagrange equations. A reasonable way to get qualitative results though is as follows - we simply take the vacuum embedding solution and plot the potential as a function of that embedding multiplied by an arbitrary constant. This gives embeddings for all $L(0)$ and all values of c . The case when we multiply by zero is of course the central maximum of the wine bottle potential. We will assume that this form is appropriate for all values of $L(0)$ or c . This then lets us plot the potential for each of our models above - the potential automatically takes the correct value at the maximum and minimum. See Fig 3.

Although this procedure is somewhat adhoc it allows a first rough understanding of the potential shape. We can see that the potential for the case of a magnetic field induced symmetry breaking is rather steep around the origin. The step function ansatz for β is gentler there and by decreasing A the curvature is further reduced.

Note that as one decreases A one must increase λ in order to keep the value of the potential when $c = 0$ equal. In the extreme low A limit the model is characterized by that potential value being much less than the characteristic scale of the running Λ . The quark condensate also grows in this limit. Comparing the blue curve and the red curve shows that varying Γ also leads to a potential with a flatter and longer period near the origin. This is encouraging since for an inflation model we would want the potential to be as flat and as extended as possible around the origin.

The lowest A curve (Orange) shows a new minimum at zero and an intermediate maximum in the potential but this is an artefact of the crude approximation being made here. Were the maximum to really form then there would be a new solution to the embedding Euler-Lagrange equation for the D7 brane - we have checked and no such solution exists. We are simply describing the non-vacuum configurations incorrectly. Fig 3 is we stress just to show qualitatively that a Higgs-like potential exists in the theory. Indeed we will see below that the time evolution of a rolling configuration depends crucially on the behaviour in the holographic directions of the description of the gauge theory which these plots are blind to. To do a better job of understanding the dynamics of these models near the origin we turn to studying the full time dependent problem of a moving D7 brane.

IV. HOLOGRAPHY OF INFLATION

A. Time Dependent Holography

We will now move on to look at time dependent evolution in the gauge theory described above. Strictly the AdS/CFT Correspondence describes our gauge theory in rigid flat space. To include inflation we will instead assume a background holographic geometry given by

$$ds^2 = \frac{1}{g_{UV}} \left[\frac{r^2}{R^2} (g_{tt} dt^2 + g_{ij} dx^i dx^j) + \frac{R^2}{r^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dw_5^2 + dw_6^2) \right], \quad (22)$$

where $g_{tt} = -1$ and $g_{ij} = a(t)^2 \delta_{ij}$. For this to be valid even as an approximation we must formally assume that $a(t)$ is growing much slower than the typical time for equilibration in the $\mathcal{N} = 4$ Yang-Mills plasma. How then can we hope to study inflation with exponential space-time growth? The formal answer is that we will study inflation driven by the flavour sector of our gauge theory. The contribution of the D7 branes to the full vacuum energy of the field theory is $1/N_c$ suppressed relative to the glue dynamics contributions. We will be able to study the theory where this perturbation to the vacuum energy drives inflation but formally in a limit where the inflation is still slow relative to the $\mathcal{N} = 4$ equilibration

time. Our inflation is that relevant for the quark fields. This provides a formal justification for what follows.

In fact for a true model of inflation one would want exponential expansion relative to all time scales in the problem. The reader can therefore choose to view (22) as a phenomenological model. In practice for the results we wish to extract - what dilaton profile will generate slow rolling behaviour - one could just work in static x_4 flat space. However, it is useful to have the damping term in the scalar equation of motion in (12) to remove late time oscillations - this provides another justification for using (22).

The DBI action in such a geometry is given by

$$S_M = S_{DBI} = \int d^4x \sqrt{-g} \mathcal{L} \quad (23)$$

where

$$\mathcal{L} = -\tilde{T}_7 R^4 \int d\rho \beta \rho^3 \sqrt{\frac{g^{00} \dot{L}^2}{(\rho^2 + L^2)^2} + 1 + L'^2} \quad (24)$$

Here $g_{\mu\nu}$ are boundary field theory metric components and $L = \omega_5(t, \rho)$ with $\omega_6 = 0$. $L' = \partial_\rho L$ and $\dot{L} = \partial_t L$. All variables $(\rho, L, t, \Gamma, \lambda)$ in the integral (24) are rescaled by R and dimensionless. We will reinstate R when needed. Note that the 4D effective action derived from the DBI action can be written in a covariant form with the correct measure.

The integral form of the 4d Lagrangian (24) may be understood as the effective Lagrangian of some field theory quantity after integration over the extra direction ρ . Since the bulk embedding dynamics is closely related to the spontaneous symmetry breaking in the boundary field theory, it is natural to associate an effective degree of freedom to an order parameter, the condensate $c(t)$. Thus we may consider (24) as an inflaton model of a composite scalar field $c(t)$ with some potential $V(c)$. The quark condensate c can be extracted numerically from the asymptotic large ρ form of the solution (19). Even though it's not straightforward to read the form of $V(c)$ from (24) it is not an obstacle to the study of inflation, since ε and p can be computed from (24) and we can apply the inflation condition (6). Furthermore our potential for the condensate has its origin in (24), so is determined in principle and not put in by hand.

We can obtain ε and p by computing the expectation value of the stress energy tensor of the 3+1d field theory from

$$\langle T^{\mu\nu} \rangle = \frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}}. \quad (25)$$

According to the AdS/CFT Correspondence the gravitational action is the Masterfield of the field theory [1-4] with the boundary values of gravitational fields playing the roles of the sources. Here we have explicitly shown the behaviour of the gravity action on the boundary met-

ric so we may simply compute. We find

$$\varepsilon(t) = \tilde{T}_7 R^4 \int d\rho \rho^3 \beta \frac{1 + L'^2}{\sqrt{1 + L'^2 - \frac{\dot{L}^2}{(\rho^2 + L^2)^2}}} - \varepsilon_0 \quad (26)$$

$$p(t) = -\tilde{T}_7 R^4 \int d\rho \rho^3 \beta \sqrt{1 + L'^2 - \frac{\dot{L}^2}{(\rho^2 + L^2)^2}} - p_0 \quad (27)$$

where we have renormalized by subtracting ε_0, p_0 , which are the values of the integrals with the asymptotic static symmetry breaking solution for L , say L_s . Examples of L_s are shown in Fig 2. There is no explicit $a(t)$ dependence in ε and p , but its information is encoded in $L(t, \rho)$ since $L(t, \rho)$ is a solution of equation of motion of (23) which includes $a(t)$ in $\sqrt{-g}$.

Note that $p(0) = -\varepsilon(0)$ if $\dot{L} = 0$. So if an initial condition has $\dot{L} \sim 0$ then the Universe will start inflating ($p < -\varepsilon/3$) regardless of an initial L' . However we know that eventually the embedding will asymptote to L_s because of the wine bottle shaped potential, which implies $\varepsilon \rightarrow 0$ and $p \rightarrow 0$. Thus we expect ε is decreasing and p is increasing with time. If p increases faster than ε then there will be a time when $\varepsilon + 3p = 0$, ending the inflation. How long it takes depends on the initial configuration $L(0, \rho)$ and the parameters of β , the dilation profile. According to small field inflation models it is natural to start with $L = 0$, which is a local maximum configuration and the symmetric minimum at high temperature. Then the inflation time or the number of e-folds (N_e) can be studied as a function of coupling β . In our setup inflation always happens to some degree and the issue is the amount of inflation.

Since we know ε and p we can proceed with the Friedmann (2) and fluid (3) equations. Let's start with the Friedman equation.

$$H^2 = \frac{1}{3}\varepsilon, \quad (28)$$

where ε is (26). This is very complicated. ε has $H(t)$ in $a(t)$ which enters in the equation of motion for L . Also the Friedmann equation is coupled to the fluid equation.

To make the computation tractable we start with the zeroth order slow rolling approximation by assuming ε is almost constant for a long enough time, namely $\varepsilon = V_0$. It gives us a simple solution for $a(t)$

$$a(t) = e^{\sqrt{\frac{V_0}{3}}t}, \quad (29)$$

where we set $a(0) = 1$. This solution must be consistent with the fluid equation, which is equivalent to the equation of motion for L . Thus we will solve the equation of motion resulting from (23) with a constant V_0 (29) and then plug the solution back into (26). If the calculated ε changes slowly enough (slow rolling), then our solution is self-consistent. In our numerical computation ε and p are always rescaled as $\varepsilon \rightarrow \varepsilon \tilde{T}_7 R^4$ and $p \rightarrow p \tilde{T}_7 R^4$ so are

dimensionless.

B. Rolling in a B field

As a first example of our formalism we will consider the case of magnetic field induced symmetry breaking. We do not expect such a model to be well suited to a realistic inflation model because the typical curvature of the potential (Fig 3) for the condensate c is large. There is no particular fine tuning in the model. We will be able to track the time evolution of the brane configuration though from a symmetric to a symmetry breaking vacuum.

We will first study the time dependence of the model in a constantly inflating Universe *i.e.* with fixed cosmological constant H . In reality H should be determined by the D7 energy density through the evolution (H depends on the choices of N_c and the 'tHooft coupling - we will show some generic numerical results). In the early inflating stages of the evolution we are most interested in, assuming a constant H is sound. For the evolution near the true vacuum H should go to zero and the brane would be expected to oscillate about the true vacuum configuration.

Most of the solutions we will show will remain highly damped in the late time regime which allows us to numerically test that the configuration indeed ends on the static vacuum D7 embedding. We will though show some results for $H = 0$, where those oscillations are observable, shortly. The high damping is due to our unphysical constant H at all time. In a full computation it would be time dependent and vanish at late time. If we could solve with a time dependent H we would find the oscillation around the true vacuum at late time as hinted in the case $H = 0$.

In particular we will start with the initial conditions

$$L(\rho, t = 0) = 0, \quad \dot{L}(\rho, t = 0) = v e^{-\rho^2}, \quad (30)$$

where $\dot{L} \equiv \partial_t L$. The initial speed and ρ dependence of this ansatz is not picked for any deep reason but is just illustrative of some initial condition that initiates the roll down to the potential minimum. We have checked that none of our results are qualitatively changed by varying for example the width of this initial condition. We will typically pick v to be very small so that the roll time from the peak of the potential is quite long. This also ensures that inflation happens at early times. The early time inflationary period in the plots below are the result of this fine tuned small initial condition and the large damping term and not a sign that the potential is particularly flat near the origin.

A slow early roll will mean we can study the dynamics more easily numerically and understand better how to lengthen that early roll period. Through the roll we must ensure that $L'(0, t) = 0$ and that $L(\infty, t) = 0$. The evolution can in fact be followed with these boundary

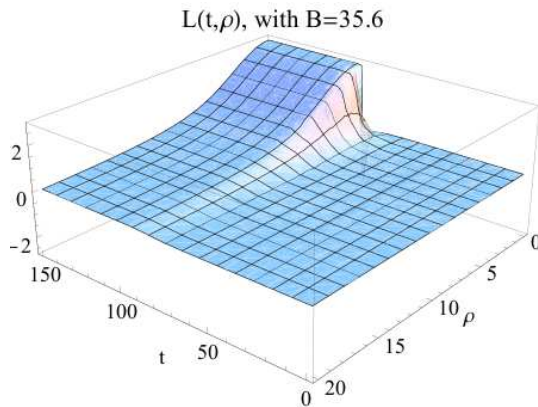


FIG. 4: We show the time evolution of the D7 brane with $H = \sqrt{70/3}$, $B = 35.6$ and initial velocity parameter $v = 0.00001$. The transition from the flat embedding (chirally symmetric phase) to the curved embedding (chirally broken phase) is apparent.

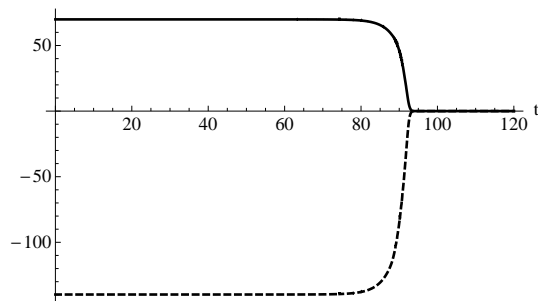


FIG. 5: $\varepsilon + 3p$ (dotted) and ε (solid) plotted against time for the time evolution of the configuration shown in Fig 4.

conditions using the in built numerical partial differential equation solver in Mathematica.

In Fig 4 we show a sample plot of the numerical evolution. The figure is for an inflating Universe with $H = \sqrt{70/3}$, $B = 35.6$. We show a three dimensional plot for the time evolution of the D7 embedding - at early times the D7 is flat, $L = 0$, but as soon as the kinetic energy of the brane begins to grow as it experiences the curvature of the potential, it rapidly transitions to the vacuum embedding with symmetry breaking.

Fig 5 is a plot of $\varepsilon + 3p$ and ε versus time. $\varepsilon(t) \approx -p(t)$ at all time. ε shows a plateau until $t \sim 80$ and changes abruptly, while $\varepsilon + 3p$ is negative. Thus our slow roll approximation is valid up to $t \sim 80$. Even though the solution after $t \sim 80$ is beyond our approximation we read off the inflation ending time as the time when $\varepsilon + 3p = 0$ as an estimate, which is $t_e \sim 93$. The end time will depends on B or H , which are related.

We stress again that we have taken the Hubble parameter H constant through the brane motion in the results just presented. With this unphysical late time damping it can be seen that the solutions precisely match on to the time independent vacuum configuration at large time.

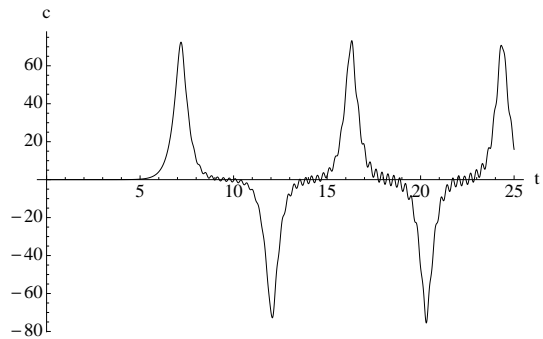


FIG. 6: We show the time evolution of the quark condensate with $H = 0$, $B = 30$ and initial velocity parameter $v = 0.00001$. The D7 brane oscillates from the flat embedding (chirally symmetric phase) to the curved embedding (chirally broken phase) above and below the axis.

To show the oscillatory behaviour one expects when the Hubble parameter is not present we can also solve for a similar configuration with $H = 0$. We show a plot in Fig 6 of the time dependence of the quark condensate c in such a scenario - c is extracted from the large ρ dependence of the solution through (19). With no damping at all the D7 moves to approximately the vacuum configuration, then overshoots, returns to the flat embedding before moving below the axis, etc. The oscillatory behaviour is clear and can be followed through many cycles. Note that there are “fine wrinkles” near $c = 0$ in this plot. These are due to a peculiarity of the magnetic field induced symmetry breaking - in particular there are an infinite set of meta-stable vacua near $c = 0$ in this theory as explored in [26]. We see their influence on this motion although they play no particular role in our analysis here. Our phenomenological dilaton profiles below do not generate such structure.

Let us briefly return to compare these simulations to what would be needed for inflation in our Universe. The energy density (23) is naively

$$\varepsilon = \tilde{T}_7 R^4 \mathbb{E}(B), \quad (31)$$

where $\mathbb{E}(B)$ is a number obtained by the numerical integration in (24). Note the choice of the magnetic field here introduced the intrinsic scale of symmetry breaking.

ε is also a measure of the symmetry breaking scale of the theory and in flat space we could set it to one as the defining scale in the problem. When we include the inflationary dynamics through the damping term in (24) the Planck scale also enters. The ratio of the Planck scale to the fourth root of ε is a free parameter. We will associate the Planck mass with some distance in the holographic ρ direction, γ_p . The physical Planck mass in the coordinates with which we compute (rescaled by R as discussed under (24)) is thus

$$m_P = \frac{\gamma_p}{R}. \quad (32)$$

It is then sensible to express the energy density in Planck units so

$$\varepsilon = \varepsilon_0 m_P^4, \quad (33)$$

where

$$\varepsilon_0 \equiv \mathbb{E}(B) \frac{1}{2(2\pi)^5 g_{UV}^2 \gamma_p^4} \frac{R^8}{\alpha'^4} = \mathbb{E}(B) \frac{2\lambda_{tH} N}{(2\pi)^3 \gamma_p^4}, \quad (34)$$

Note this α' is independent as it should be since $R^8/\alpha'^4 = (4\pi g_{UV}^2 N)^2$. $\lambda_{tH} \equiv g_{UV}^2 N$

The roll times \tilde{t}_e we have computed are in dimensionless units and should be written in Planckian units too.

$$t_e m_P = R \tilde{t}_e m_P = \gamma_p \tilde{t}_e. \quad (35)$$

The scale factor in the metric is therefore given by

$$a(t) = e^{Ht} = e^{\sqrt{\frac{8\pi\varepsilon_0}{3}} \gamma_p \tilde{t}_e}. \quad (36)$$

Our simulations have been with $\sqrt{\frac{8\pi\varepsilon_0}{3}} \gamma_p = \sqrt{70/3}$. Note one can realize this for any value of ε_0 - once ε_0 is fixed we can choose γ_p by setting the gauge theory parameters such as λ_{tH} and N_c .

We conclude this section with the comment that it is quite remarkable that we can compute so straight forwardly the time evolution of a strongly coupled gauge theory!

C. Towards an Inflationary Dilaton Profile

Our next goal is to look to extend the period of slow roll in the holographic models by phenomenologically changing the dilaton profile or gauge coupling's running. This is the equivalent of making the usual inflaton potential flatter around the origin.

To examine this analytically let us first consider the roll in static space $H = 0$. We linearize the equation of motion about the initial symmetry preserving configuration $L = 0$

$$\ddot{L} = 3\rho^3 L' + \rho^4 L'' + \rho^4 \frac{\beta'_0}{\beta_0} L' - \rho^3 \frac{\beta'_0}{\beta_0} L, \quad (37)$$

where β_0 is the coupling at $L = 0$ (for the step function form of the running we have $\beta_0 = A+1 - A \tanh[\Gamma(\rho-\lambda)]$ and $\beta'_0 = -A\Gamma \text{sech}[\Gamma(\lambda-\rho)]^2$). If we consider a truly flat configuration so $L' = L'' = 0$ then only the last term on the right hand side contributes. Clearly the acceleration of the brane is localized around the point in ρ where there is a significant change in the coupling value. We show that this is indeed how the motion proceeds in Fig 7 where we show a plot of the early time evolution of the brane in the presence of a step in the value of the dilaton. A bump grows around the point in ρ where the dilaton changes. We also plot the full evolution with a

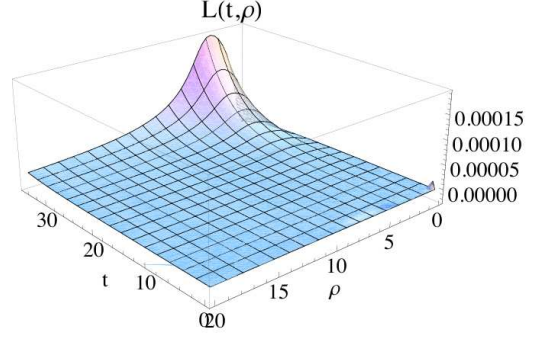


FIG. 7: The early time evolution of the D7 brane in the presence of a dilaton step with $A = 3$, $\lambda = 3.24$, $\Gamma = 1$ and for the initial condition $v = 0.00001$. $H = \sqrt{70/3}$. The acceleration occurs around the step.

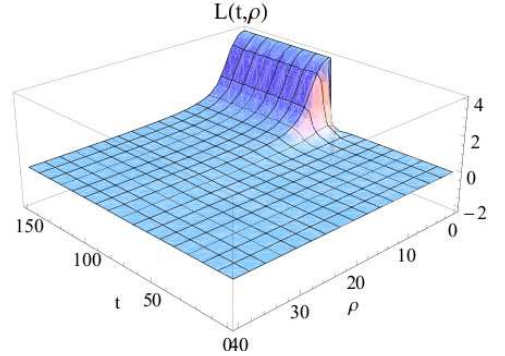


FIG. 8: The full time evolution of the D7 brane in the presence of a dilaton step with $A = 3$, $\lambda = 3.24$, $\Gamma = 1$ and for the initial condition $v = 0.00001$. $H = \sqrt{70/3}$.

cosmological damping parameter $H = \sqrt{70/3}$ showing the configuration move to the true vacuum in Fig 8.

For the step configuration with large Γ , so we can consider it to be a sharp step, we have

$$\ddot{L}(\rho) \sim \lambda^3 \frac{A\Gamma}{A+1} L(\lambda) \delta(\rho - \lambda), \quad (38)$$

naively lowering A or Γ will reduce the acceleration. In fact though to keep the difference in the potential between the symmetric and symmetry breaking configurations equal as we reduce A or Γ we must increase λ . For the numerical values in Fig 3, increasing λ wins so $\frac{A\Gamma}{1+A} \lambda^3$ increases when A decreases.

This analysis is overly naive though because as the D7 brane evolves, as shown in Fig 7, L' and L'' will become important. Around the localized peak of the bump where $L' = 0$ L'' is negative and the equation of motion will be

$$\ddot{L}(\rho) \sim \left(\lambda^3 \frac{A\Gamma}{A+1} L(\lambda) + \lambda^4 L''(\lambda) \right) \delta(\rho - \lambda). \quad (39)$$

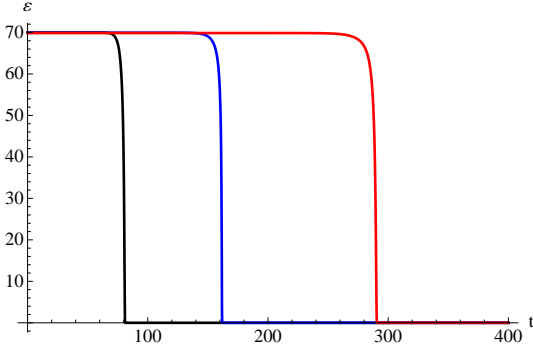


FIG. 9: Plots of ε vs t for the three configurations $[A = 3, \Gamma = 1, \lambda = 3.24]$, $[A = 1.8, \Gamma = 1, \lambda = 4.325]$ and $[A = 1.8, \Gamma = 0.5, \lambda = 5.882]$. $v = 0.00001$. $H = \sqrt{70/3}$.

Since L'' is negative the latter term will slow the acceleration in the $A \rightarrow 0, \lambda \rightarrow \infty$ limit. Thus we conclude that for models with fixed H those with a larger value of λ will provide a longer period of inflation. We want to stress that this conclusion is completely dependent on the holographic description through evolution associated with the ρ direction - we are learning about the role of the strong interaction dynamics. One certainly can not, for example, deduce the motion from the simplistic approximate potentials we displayed in Fig 3. More generally we note that the holographic dependence on ρ introduces considerably more complication to the evolution and explicit simulation is required.

It is clear that if we wish to prolong the early time roll period we need to reduce the rate of change of the dilaton and push λ far above the vacuum energy H . There are two ways we can do that within our ansatz (21) - we can reduce A for a fixed Γ so that the step is smaller. We would expect this to increase the roll time. Also for fixed A we can decrease Γ so the change in A occurs over a larger ρ range. This reduces the dilaton derivative but spreads the region of ρ over which the change is occurring so there could be no net change to the total rate of acceleration - we will need to test this case numerically.

In Fig 9 we plot ε for the three different sets of parameters in β . Like Fig 5, $\varepsilon + 3p$ is always negative before it suddenly vanishes. Even though our computation is not valid when ε starts changing fast we choose to take the time when $\varepsilon + 3p$ vanish as the end time of inflation, t_e . Since our purpose is to compare the inflation time for different dilaton profiles qualitatively, this approximation will not make a difference.

We will now make a comparison of the roll time for a number of different dilaton step profiles. In each case the difference in vacuum energy between the symmetric and vacuum symmetry breaking configuration is the same. We also use the same initial velocity perturbation for the D7 brane (30). As an example of the differences we plot the energy density ε against time in Fig 9 and the condensate c against time for three configurations in Fig 10. First compare the curves for the dilaton parameters

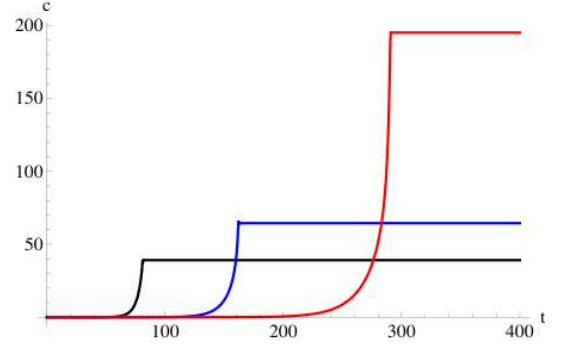


FIG. 10: The condensate c vs t , for the three configurations $[A = 3, \Gamma = 1, \lambda = 3.24]$, $[A = 1.8, \Gamma = 1, \lambda = 4.325]$ and $[A = 1.8, \Gamma = 0.5, \lambda = 5.882]$. $v = 0.00001$. $\kappa = \sqrt{70/3}$.

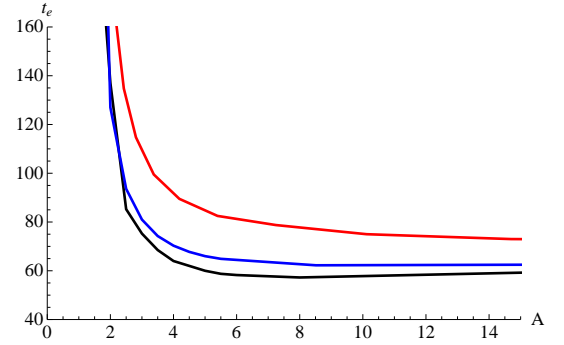


FIG. 11: t_e against A for a sequence of values of Γ s: 1.5(Black), 1(Blue), 0.5(Red). $v = 0.00001$. $H = \sqrt{70/3}$.

$[A = 3, \Gamma = 1, \lambda = 3.24]$ and $[A = 1.8, \Gamma = 1, \lambda = 4.325]$. Decreasing A indeed increases the time the configuration takes to reach the tipping point to the true vacuum. Next we can change Γ to try to further increase the roll time - the final curve is for the configuration $[A = 1.8, \Gamma = 0.5, \lambda = 5.882]$ and indeed we find a further lengthening of the inflationary period.

Finally we show these trends in more detail in Fig 11 where we plot t_e against A for a sequence of values of Γ (1.5, 1, 0.5). There is a clear trend for decreasing both A and Γ increasing the roll time. As this roll time increases the parameters become more fine tuned reflecting the fine tuning we are making in the effective potential for c , ie the usual fine tuning in inflation.

This analysis has been performed again in a heavily damped scenario which as discussed in (36) assumes a growing value of N in the gauge theory as the energy density is reduced relative to the Planck scale. We choose this regime primarily for computational convenience. Having high damping removes late time oscillations of the D7 motion. It also extends the roll time making changes in that roll time more easily apparent. To demonstrate that the effects we have observed are still present at lower values of the damping we finally display the behaviour of the condensate against time for the case

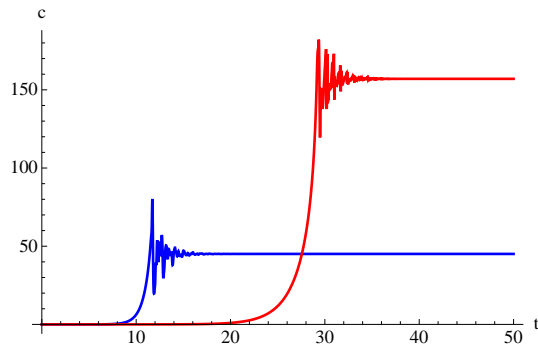


FIG. 12: t_e against A for two cases sequence of values. Blue: $[A = 27.3, \Gamma = 0.5, \lambda = 0.25]$, Red: $[A = 2, \Gamma = 0.5, \lambda = 5.326]$. $v = 0.00001$. $H = \sqrt{1/3}$.

$H = \sqrt{1/3}$ and for steps with two different values of step height A in Fig 12. The late time oscillations are now apparent but the increase of the roll time with decreasing A is also maintained.

V. DISCUSSION

Inspired by small field inflation models, we have used holography to study out of equilibrium dynamics in a strongly coupled gauge theory with chiral symmetry breaking. We have shown that we can extract an approximate effective potential for the quark condensate. We have then been able to explicitly follow the roll of a configuration from the symmetric potential maximum to the symmetry breaking potential minimum. We first performed this computation in a model with chiral symmetry breaking induced by a magnetic field. While the configuration lies near the potential maximum it generates inflation. Formally we worked in a probe limit that requires that the rate of inflation is small relative to the typical relaxation time of the background $\mathcal{N} = 4$ gauge dynamics.

We then studied a holographic model of a large N_c gauge theory with an arbitrary running of the gauge coupling. It is a model in the spirit of AdS/QCD since we do not backreact the geometry to the dilaton profile. The dilaton profile we chose to study means the gauge theory has a strongly coupled conformal UV and deforms to an IR conformal theory with a higher value of the gauge coupling. That change in the coupling triggers chiral symmetry breaking for the quarks. We have found that when the change in the coupling is fine tuned to be small or slow running the theory can give rise to inflationary dynamics. An important contribution to this slowing of the roll was made by the D7 dynamics in the radial direction of the geometry through (39) - this suggests that the strong dynamics of these gauge theories encourages inflation. Are there broad lessons to be learnt here about strongly coupled gauge theories or is this an oddity of a particular AdS set up?

There is a growing belief that many asymptotically free gauge theories indeed give rise to strongly coupled conformal regimes. Seiberg's dualities for $\mathcal{N} = 1$ supersymmetric QCD [27] were the first hint - they show that SQCD flows to a non-trivial IR conformal theory in the range $N_c + 1 < N_f < 3N_c$. For much of that range the UV degrees of freedom are strongly coupled. Near $N_f = 3N_c$ and at large N_c these phases match onto the perturbative Banks Zaks fixed points [28]. Banks Zaks fixed points also exist in non-supersymmetric theories and it is reasonable to expect that IR conformal fixed points also exist for a considerable range of N_f in those theories (see [29], [30] for some speculation about these theories). Theories with higher dimension representation matter fields would also be expected to generate strongly coupled IR fixed points. Recently there has begun to be lattice simulations of QCD with varying unquenched quark flavours [31, 32] and higher dimensional representation matter present [33, 34] - there is certainly encouragement in these results for the view that IR conformal theories exist in some of these cases.

If we believe that such fixed points are fairly common then we can imagine several ways for how to construct a theory with the profile for the running coupling we have studied in this paper. For example, one could begin with an $SU(N_c)$ theory with sufficient non-fundamental matter to place it at a strongly coupled fixed point. If N_c is appropriately large then we can also add fundamental quark multiplets as a perturbation - their contributions to the beta function coefficients will be N_f/N_c suppressed and hence they will most likely generate just a small change in the fixed point's coupling value. The fundamental quarks though can be used to dial a profile for the running coupling if they can be sequentially decoupled. For example, if they are vector-like one could just put in masses to adjust the running at the order N_f/N_c shifting the IR theory from one conformal point to another. This realizes the running we described above. It is not obvious how chiral symmetry breaking might happen in this scenario though. Usually one imagines an NJL-model-like critical coupling for chiral symmetry breaking [25]. One would need to tune the shift in the coupling to cross that critical value for the fundamental quarks. Naively higher dimension representation quarks might also be expected to condense at the same point or at a lower value of the coupling though.

Another possibility, which is sometimes discussed [35], is that pure glue gauge theories in fact have an IR fixed point for the coupling. One could imagine perturbing that fixed point with some fundamental matter and again, by appropriate decoupling, change the running to match that we seek. The pure glue fixed point would again need to lie just above the critical coupling for chiral symmetry breaking.

Finally the other possibility we are led to is walking dynamics [36, 37]. A theory with fundamental quarks that is approaching an IR fixed point where the coupling is tuned just above the critical coupling might spend many

RG decays at strong coupling but without triggering chiral symmetry breaking before finally reaching the critical value at a low scale. Equally the coupling might cross the critical value at a high scale but be so close to the critical value from above that the quarks' dynamical mass is at a much lower scale so they don't decouple and the fixed point is maintained. Such a theory would naturally realize both the small A and small Γ limit of our coupling ansatz, both of which led towards inflation. Walking is of course proposed as a solution of the mini-hierarchy problem in technicolour theories of electroweak symmetry breaking [38, 39]. It is certainly intriguing if the fine tuning already used to solve that problem also generates inflationary dynamics.

In conclusion it certainly seems possible that a range

of asymptotically free gauge theories might realize the behaviour we have seen and generate effective dynamics that encourages inflation. The need for intrinsic fine tuning, as usual in small field inflation, seems unavoidable though.

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