Künstliche Intelligenz – Übung 10

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Organizational

Homework 9

Homework 10

Misc: Questions, Anecdotes & etc

Organizational

Personal information

my email address: marius.frinken@fau.de
PGP encrypted mails are preferred!
my PGP fingerprint:
`F4BD 7ED4 96A5 9BA6 9FD6 901C 1EEC 9B1B 8CD5 3DA1`

Evaluation

Get your **TAN** and use it here: https://eva.fau.de/
Please fill out the form until **26th January 2019, 12:00 (noon)**

My proper first name is Marius, don't be fooled by your TAN ;)

KALAH Tournament

see https://fsi.cs.fau.de/forum/thread/16932-Kalah-Tournament

Homework 9

Problems 9.2 - common errors

WRONG

$$\cfrac{\cfrac{\cfrac{[P \wedge Q]^1}{P}\,\wedge\mathrm{E}_\mathrm{l}^1 \qquad \cfrac{[P \wedge Q]^1}{Q}\,\wedge\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,\vee\mathrm{I}}{}\,\to\mathrm{I}^1$$

Problems 9.2 - common errors

WRONG

$$\cfrac{\cfrac{\cfrac{[P \wedge Q]^1}{P}\,\wedge\mathrm{E}_\mathrm{l}^1 \qquad \cfrac{[P \wedge Q]^1}{Q}\,\wedge\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,\vee\mathrm{I}}{}\,\to\mathrm{I}^1$$

RIGHT

$$\cfrac{\cfrac{\cfrac{[P \wedge Q]^1}{P}\,\wedge\mathrm{E}_\mathrm{l}^1 \qquad \cfrac{[P \wedge Q]^1}{Q}\,\wedge\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,\vee\mathrm{I}}{}\,\to\mathrm{I}^1$$

Problems 9.2 - common errors

WRONG

$$\cfrac{\cfrac{\cfrac{[P \wedge Q]^1}{P}\,{\wedge}\mathrm{E}_\mathrm{l}^1 \qquad \cfrac{[P \wedge Q]^1}{Q}\,{\wedge}\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,{\vee}\mathrm{I}}{\quad}\Rightarrow\mathrm{I}^1$$

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$$\cfrac{\cfrac{\cfrac{[P \wedge Q]^1}{P}\,{\wedge}\mathrm{E}_\mathrm{l}^1 \qquad \cfrac{[P \wedge Q]^1}{Q}\,{\wedge}\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,{\vee}\mathrm{I}}{\quad}\Rightarrow\mathrm{I}^1$$

$$\cfrac{\begin{matrix}[P]^2 \\ Q\end{matrix}}{(P \Rightarrow Q)} \Rightarrow I^2$$

Problems 9.2 - common errors

WRONG

$$\dfrac{\dfrac{\dfrac{[P \wedge Q]^1}{P}\,\wedge\mathrm{E}_\mathrm{l}^1 \qquad \dfrac{[P \wedge Q]^1}{Q}\,\wedge\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,\vee\mathrm{I}}{}\,\Rightarrow\mathrm{I}^1$$

$$\dfrac{\begin{matrix}[P]^2 \\ Q\end{matrix}}{(P \Rightarrow Q)} \Rightarrow I^2$$

RIGHT

$$\dfrac{\dfrac{\dfrac{[P \wedge Q]^1}{P}\,\wedge\mathrm{E}_\mathrm{l}^1 \qquad \dfrac{[P \wedge Q]^1}{Q}\,\wedge\mathrm{E}_\mathrm{r}^1}{P \vee Q}\,\vee\mathrm{I}}{}\,\Rightarrow\mathrm{I}^1$$

$$\dfrac{\begin{matrix}[P]^2 \\ Q\end{matrix}}{(P \Rightarrow Q)} \Rightarrow I^2$$

Problem 9.3

(show solution)

Conclusion: it's just simple transformations / clever usage of definitions

Homework 10

Recap Tableaux I

Test Calculi: Tableaux and Model Generation

- Idea: instead of showing $\emptyset \vdash Th$, show $\neg Th \vdash trouble$ (use $\bot$ for trouble)
- **Example 5.10.** Tableau Calculi try to construct models.

Tableau Refutation (Validity)	Model generation (Satisfiability)
$\models P \wedge Q \Rightarrow Q \wedge P$	$\models P \wedge (Q \vee \neg R) \wedge \neg Q$
$P \wedge Q \Rightarrow Q \wedge P^F$ $P \wedge Q^T$ $Q \wedge P^F$ P^T Q^T $P^F \mid Q^F$ $\bot \mid \bot$	$P \wedge (Q \vee \neg R) \wedge \neg Q^T$ $P \wedge (Q \vee \neg R)^T$ $\neg Q^T$ Q^F P^T $Q \vee \neg R^T$ $Q^T \mid \neg R^T$ $\bot \mid R^F$
No Model	Herbrand Model $\{P^T, Q^F, R^F\}$ $\varphi := \{P \mapsto T, Q \mapsto F, R \mapsto F\}$

Algorithm: Fully expand all possible tableaux, (no rule can be applied)

- ▶ Satisfiable, iff there are open branches (correspond to models)

Recap Tableaux II

Analytical Tableaux (Formal Treatment of $\mathcal{T}_0$)

- formula is analyzed in a tree to determine satisfiability
- branches correspond to valuations (models)
- one per connective

$$\frac{\mathbf{A} \wedge \mathbf{B}^{\mathsf{T}}}{\begin{array}{c}\mathbf{A}^{\mathsf{T}}\\ \mathbf{B}^{\mathsf{T}}\end{array}}\mathcal{T}_0\wedge \qquad \frac{\mathbf{A} \wedge \mathbf{B}^{\mathsf{F}}}{\mathbf{A}^{\mathsf{F}} \mid \mathbf{B}^{\mathsf{F}}}\mathcal{T}_0\vee \qquad \frac{\neg\mathbf{A}^{\mathsf{T}}}{\mathbf{A}^{\mathsf{F}}}\mathcal{T}_0{\neg}^{\mathsf{T}} \qquad \frac{\neg\mathbf{A}^{\mathsf{F}}}{\mathbf{A}^{\mathsf{T}}}\mathcal{T}_0{\neg}^{\mathsf{F}} \qquad \frac{\begin{array}{c}\mathbf{A}^{\alpha}\\ \mathbf{A}^{\beta}\end{array} \quad \alpha \neq \beta}{\bot}\mathcal{T}_0\mathrm{cut}$$

- Use rules exhaustively as long as they contribute new material
- **Definition 5.11.** Call a tableau saturated, iff no rule applies, and a branch closed, iff it ends in $\bot$, else open. (open branches in saturated tableaux yield models)
- **Definition 5.12 ($\mathcal{T}_0$-Theorem/Derivability).** $\mathbf{A}$ is a $\mathcal{T}_0$-theorem ($\vdash_{\mathcal{T}_0} \mathbf{A}$), iff there is a closed tableau with $\mathbf{A}^{\mathsf{F}}$ at the root.
  $\Phi \subseteq wff_o(\mathcal{V}_o)$ derives $\mathbf{A}$ in $\mathcal{T}_0$ ($\Phi \vdash_{\mathcal{T}_0} \mathbf{A}$), iff there is a closed tableau starting with $\mathbf{A}^{\mathsf{F}}$ and Φ^{T}.

Recap Tableaux III

Derived Rules of Inference

- **Definition 5.18.** Let $\mathcal{C}$ be a calculus, a rule of inference $\frac{\mathbf{A}_1 \quad \cdots \quad \mathbf{A}_n}{\mathbf{C}}$ is called a derived inference rule in $\mathcal{C}$, iff there is a $\mathcal{C}$-proof of $\mathbf{A}_1, \ldots, \mathbf{A}_n \vdash \mathbf{C}$.
- **Definition 5.19.** We have the following derived rules of inference

$$\frac{\mathbf{A} \Rightarrow \mathbf{B}^{\mathsf{T}}}{\mathbf{A}^{\mathsf{F}} \mid \mathbf{B}^{\mathsf{T}}} \qquad \frac{\mathbf{A} \Rightarrow \mathbf{B}^{\mathsf{F}}}{\begin{array}{c}\mathbf{A}^{\mathsf{T}} \\ \mathbf{B}^{\mathsf{F}}\end{array}} \qquad \frac{\begin{array}{c}\mathbf{A}^{\mathsf{T}} \\ \mathbf{A} \Rightarrow \mathbf{B}^{\mathsf{T}}\end{array}}{\mathbf{B}^{\mathsf{T}}}$$

$$\frac{\mathbf{A} \vee \mathbf{B}^{\mathsf{T}}}{\mathbf{A}^{\mathsf{T}} \mid \mathbf{B}^{\mathsf{T}}} \qquad \frac{\mathbf{A} \vee \mathbf{B}^{\mathsf{F}}}{\begin{array}{c}\mathbf{A}^{\mathsf{F}} \\ \mathbf{B}^{\mathsf{F}}\end{array}} \qquad \frac{\mathbf{A} \Leftrightarrow \mathbf{B}^{\mathsf{T}}}{\begin{array}{c|c}\mathbf{A}^{\mathsf{T}} & \mathbf{A}^{\mathsf{F}} \\ \mathbf{B}^{\mathsf{T}} & \mathbf{B}^{\mathsf{F}}\end{array}} \qquad \frac{\mathbf{A} \Leftrightarrow \mathbf{B}^{\mathsf{F}}}{\begin{array}{c|c}\mathbf{A}^{\mathsf{T}} & \mathbf{A}^{\mathsf{F}} \\ \mathbf{B}^{\mathsf{F}} & \mathbf{B}^{\mathsf{T}}\end{array}}$$

$$\begin{array}{c}
\mathbf{A}^{\mathsf{T}} \\
\mathbf{A} \Rightarrow \mathbf{B}^{\mathsf{T}} \\
\neg \mathbf{A} \vee \mathbf{B}^{\mathsf{T}} \\
\neg(\neg\neg \mathbf{A} \wedge \neg \mathbf{B})^{\mathsf{T}} \\
\neg\neg \mathbf{A} \wedge \neg \mathbf{B}^{\mathsf{F}} \\
\begin{array}{c|c}
\neg\neg \mathbf{A}^{\mathsf{F}} & \neg \mathbf{B}^{\mathsf{F}} \\
\neg \mathbf{A}^{\mathsf{T}} & \mathbf{B}^{\mathsf{T}} \\
\mathbf{A}^{\mathsf{F}} & \\
\bot &
\end{array}
\end{array}$$

Example

(see blackboard)

CNF

Conjunctive Normal Form:

Formula: $\Phi = C_0 \wedge C_1 \wedge ... \wedge C_n$

Clause: $C_i = L_0 \vee L_1 \vee ... \vee L_m$

Literal: $L_i \in \{\neg X, X\}$ where X is a variable

Recap Resolution I

Another Test Calculus: Resolution

- **Definition 5.26 (Resolution Calculus).** The resolution calculus operates a clause sets via a single inference rule:

$$\frac{P^{\mathsf{T}} \vee \mathbf{A} \quad P^{\mathsf{F}} \vee \mathbf{B}}{\mathbf{A} \vee \mathbf{B}}$$

  This rule allows to add the clause below the line to a clause set which contains the two clauses above.

- **Definition 5.27 (Resolution Refutation).** Let S be a clause set, then we call a $\mathcal{R}$ derivation $\mathcal{D}\colon S \vdash_{\mathcal{R}} \square$ resolution refutation.
- **Definition 5.28 (Resolution Proof).** We call a resolution refutation of $\mathit{CNF}^0(\mathbf{A}^{\mathsf{F}})$ a resolution proof for $\mathbf{A} \in \mathit{wff}_o(\mathcal{V}_o)$.

Recap Resolution II

Clause Normal Form Transformation (A calculus)

- **Definition 5.29.** A clause is a disjunction of literals. We will use $\Box$ for the empty disjunction (no disjuncts) and call it the empty clause.
- **Definition 5.30.** We will often write a clause set $\{C_1, \ldots, C_n\}$ as $C_1; \ldots; C_n$, use $S; T$ for the union of the clause sets S and T, and $S; C$ for the extension by a clause C.
- **Definition 5.31 (Transformation into Clause Normal Form).** The CNF transformation calculus CNF^0 consists of the following four inference rules on sets of labeled formulae.

$$\frac{\mathbf{C} \vee (\mathbf{A} \vee \mathbf{B})^\mathsf{T}}{\mathbf{C} \vee \mathbf{A}^\mathsf{T} \vee \mathbf{B}^\mathsf{T}} \quad \frac{\mathbf{C} \vee (\mathbf{A} \vee \mathbf{B})^\mathsf{F}}{\mathbf{C} \vee \mathbf{A}^\mathsf{F}; \mathbf{C} \vee \mathbf{B}^\mathsf{F}} \qquad \frac{\mathbf{C} \vee \neg \mathbf{A}^\mathsf{T}}{\mathbf{C} \vee \mathbf{A}^\mathsf{F}} \quad \frac{\mathbf{C} \vee \neg \mathbf{A}^\mathsf{F}}{\mathbf{C} \vee \mathbf{A}^\mathsf{T}}$$

- **Definition 5.32.** We write $\mathit{CNF}^0(\mathbf{A}^\alpha)$ for the set of all clauses derivable from $\mathbf{A}^\alpha$ via the rules above.

Recap Resolution III

Derived Rules of Inference

- **Definition 5.33.** Let $\mathcal{C}$ be a calculus, a rule of inference $\frac{\mathbf{A}_1 \quad \ldots \quad \mathbf{A}_n}{\mathbf{C}}$ is called a derived inference rule in $\mathcal{C}$, iff there is a $\mathcal{C}$-proof of $\mathbf{A}_1, \ldots, \mathbf{A}_n \vdash \mathbf{C}$.

- **Example 5.34.**

$$\dfrac{\dfrac{\dfrac{\mathbf{C} \vee (\mathbf{A} \Rightarrow \mathbf{B})^{\mathsf{T}}}{\mathbf{C} \vee (\neg \mathbf{A} \vee \mathbf{B})^{\mathsf{T}}}}{\mathbf{C} \vee \neg \mathbf{A}^{\mathsf{T}} \vee \mathbf{B}^{\mathsf{T}}}}{\mathbf{C} \vee \mathbf{A}^{\mathsf{F}} \vee \mathbf{B}^{\mathsf{T}}} \quad \leadsto \quad \dfrac{\mathbf{C} \vee (\mathbf{A} \Rightarrow \mathbf{B})^{\mathsf{T}}}{\mathbf{C} \vee \mathbf{A}^{\mathsf{F}} \vee \mathbf{B}^{\mathsf{T}}}$$

- Others:

$$\dfrac{\mathbf{C} \vee (\mathbf{A} \Rightarrow \mathbf{B})^{\mathsf{T}}}{\mathbf{C} \vee \mathbf{A}^{\mathsf{F}} \vee \mathbf{B}^{\mathsf{T}}} \quad \dfrac{\mathbf{C} \vee (\mathbf{A} \Rightarrow \mathbf{B})^{\mathsf{F}}}{\mathbf{C} \vee \mathbf{A}^{\mathsf{T}}; \mathbf{C} \vee \mathbf{B}^{\mathsf{F}}} \qquad \dfrac{\mathbf{C} \vee \mathbf{A} \wedge \mathbf{B}^{\mathsf{T}}}{\mathbf{C} \vee \mathbf{A}^{\mathsf{T}}; \mathbf{C} \vee \mathbf{B}^{\mathsf{T}}} \quad \dfrac{\mathbf{C} \vee \mathbf{A} \wedge \mathbf{B}^{\mathsf{F}}}{\mathbf{C} \vee \mathbf{A}^{\mathsf{F}} \vee \mathbf{B}^{\mathsf{F}}}$$

Recap Resolution IV

Example: Proving Axiom **S**

- **Example 5.35.** Clause Normal Form transformation

$$\frac{\frac{\frac{(P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R^{\mathsf{F}}}{P \Rightarrow Q \Rightarrow R^{\mathsf{T}}; (P \Rightarrow Q) \Rightarrow P \Rightarrow R^{\mathsf{F}}}}{P^{\mathsf{F}} \vee (Q \Rightarrow R)^{\mathsf{T}}; P \Rightarrow Q^{\mathsf{T}}; P \Rightarrow R^{\mathsf{F}}}}{P^{\mathsf{F}} \vee Q^{\mathsf{F}} \vee R^{\mathsf{T}}; P^{\mathsf{F}} \vee Q^{\mathsf{T}}; P^{\mathsf{T}}; R^{\mathsf{F}}}$$

$$CNF = \{P^{\mathsf{F}} \vee Q^{\mathsf{F}} \vee R^{\mathsf{T}}\,,\ P^{\mathsf{F}} \vee Q^{\mathsf{T}}\,,\ P^{\mathsf{T}}\,,\ R^{\mathsf{F}}\}$$

- **Example 5.36.** Resolution Proof

1	$P^{\mathsf{F}} \vee Q^{\mathsf{F}} \vee R^{\mathsf{T}}$	initial
2	$P^{\mathsf{F}} \vee Q^{\mathsf{T}}$	initial
3	P^{T}	initial
4	R^{F}	initial
5	$P^{\mathsf{F}} \vee Q^{\mathsf{F}}$	resolve 1.3 with 4.1
6	Q^{F}	resolve 5.1 with 3.1
7	P^{F}	resolve 2.2 with 6.1
8	$\square$	resolve 7.1 with 3.1

Implicit Rule Here:

You need to swap **C** from 5.31 or **A or B** from 5.26 with $\bot$ and then use this "rule": $(\bot \vee A) \Leftrightarrow A$, in order to actually work with resolution.

Example 5.26:(black board)

Example

(see blackboard)

Problem 10.1

(have a look at the assignment)
"rewriting the formulae using equivalences until you arrive at an obvious tautology" $\Rightarrow$ see the forum: `https://fsi.cs.fau.de/forum/thread/16935-Exercise-10-1-Tips-for-Rewriting`

DO NOT FORGET TO ANSWER: "Can you identify any advantages or disadvantage of the calculi, and in which situations?"

Problem 10.2

(have a look at the assignment)

see the forum at https://fsi.cs.fau.de/forum/thread/16955-10-2-NP-Complete

YOU ONLY HAVE TO PROVE NP-HARDNESS!

Problem 10.2

Hint 2

Any problem that subsumes the SAT problem is correspondingly NP ~~-complete~~ **-hard** as well ("subsumes" in the sense of: The SAT problem can be reduced to the given problem in polynomial time).

Problem 10.2

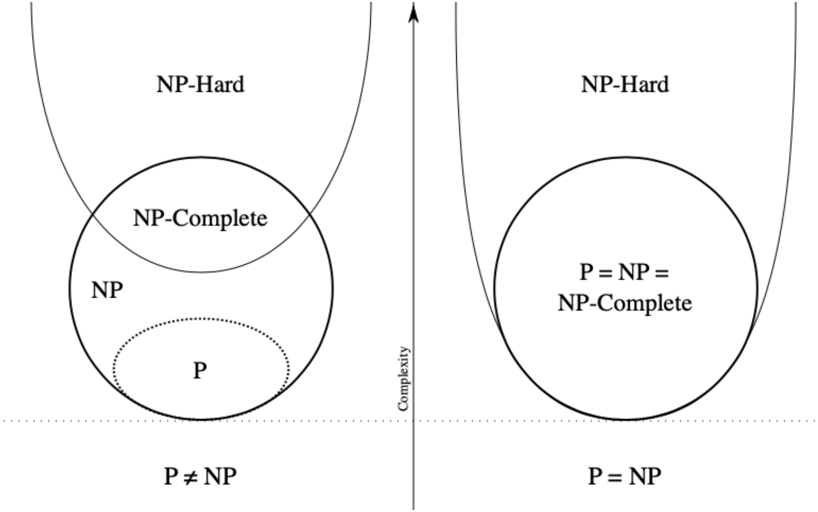
Hint 2

Any problem that subsumes the SAT problem is correspondingly NP ~~-complete~~ **-hard** as well ("subsumes" in the sense of: The SAT problem can be reduced to the given problem in polynomial time).

Now in plain English

Assume you have a solver for SAT. Your solver can be divided in two parts **A** and **B**. If you show that what **B** does is **polynomial bound** e.g. only depends linearly on the input to your solver, then what **A** does has to be **NP-Hard**.

NP-what?


NP-Hard
NP-Complete
NP
P
Complexity
P ≠ NP
NP-Hard
P = NP =
NP-Complete
P = NP

Misc: Questions, Anecdotes & etc

Questions?

I have prepared something about NP-Completeness and also we could go over 9.3 in detail, if you like...