Künstliche Intelligenz – Übung 9

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Organizational

Homework 8

Homework 9

Misc: Questions, Anecdotes & etc

Organizational

Personal information

my email address: marius.frinken@fau.de

PGP encrypted mails are preferred!

my PGP fingerprint:

`F4BD 7ED4 96A5 9BA6 9FD6 901C 1EEC 9B1B 8CD5 3DA1`

Evaluation

Get your **TAN** and use it here: https://eva.fau.de/
Please fill out the form until **26th January 2019, 12:00 (noon)**

My proper first name is Marius, don't be fooled by your TAN ;)

Homework 8

Problems 8.1, 8.2. and 8.3

(have a look at the official solution)

Homework 9

Problem 9.1

1. Show that all logical binary connectives can be expressed by the `nor` connective:
   Truth-tables !
2. rewrite $P \vee \neg P$ with it

Recap Logic and Calculus

Logic: Basic Concepts

- **Representing Knowledge:**
  - **Definition 1.2.** Syntax: What are legal statements (formulas) **A** in the logic?
  - **Example 1.3.** "W" and "$W \Rightarrow S$". ($W \widehat{=}$ *Wumpus is here*, $S \widehat{=}$ *it stinks*)
  - **Definition 1.4.** Semantics: Which formulas **A** are true under which assignment φ, written $\varphi \models \mathbf{A}$?
  - **Example 1.5.** If $\varphi := \{W \mapsto \mathsf{T}, S \mapsto \mathsf{F}\}$, then $\varphi \models W$ but $\varphi \not\models (W \Rightarrow S)$.
  - Intuition: Knowledge about the state of the world is described by formulass, interpretations evaluate them in the current world (they should turn out true!)
- **Reasoning about Knowledge:**
  - **Definition 1.6.** Entailment: Which **B** are entailed by **A**, written $\mathbf{A} \models \mathbf{B}$, meaning that, for all φ with $\varphi \models \mathbf{A}$, we have $\varphi \models \mathbf{B}$? E.g., $P \wedge (P \Rightarrow Q) \models Q$.
  - Intuition: Entailment $\widehat{=}$ ideal outcome of reasoning, everything that we can possibly conclude. e.g. determine Wumpus position as soon as we have enough information
  - **Definition 1.7.** Deduction: Which statements **B** can be derived from **A** using a set $\mathcal{C}$ of inference rules (a calculus), written $\mathbf{A} \vdash_{\mathcal{C}} \mathbf{B}$?
  - **Example 1.8.** If $\mathcal{C}$ contains $\frac{\mathbf{A} \quad \mathbf{A} \Rightarrow \mathbf{B}}{\mathbf{B}}$ then $P, P \Rightarrow Q \vdash_{\mathcal{C}} Q$
  - Intuition: Deduction $\widehat{=}$ process in an actual computer trying to reason about entailment. E.g. a mechanical process attempting to determine Wumpus position.

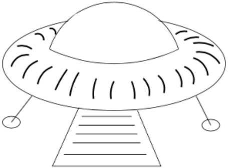
Problems? Confused?

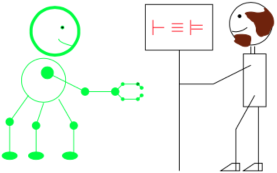
I highly recommend to read AND UNDERSTAND section 11.3 (pp 290 -298) from the slides!

The Miracle of Logic

Soundness and Completeness

- **Definition 3.15.** Let $\mathcal{S} := \langle \mathcal{L}, \mathcal{K}, \models \rangle$ be a logical system, then we call a calculus $\mathcal{C}$ for $\mathcal{S}$
  - sound (or correct), iff $\mathcal{H} \models \mathbf{A}$, whenever $\mathcal{H} \vdash_{\mathcal{C}} \mathbf{A}$, and
  - complete, iff $\mathcal{H} \vdash_{\mathcal{C}} \mathbf{A}$, whenever $\mathcal{H} \models \mathbf{A}$.
- Goal: $\vdash \mathbf{A}$ iff $\models \mathbf{A}$ (provability and validity coincide)
  - To TRUTH through PROOF (CALCULEMUS [Leibniz ~1680])





Recap Natural Deduction

Calculi: Natural Deduction ($\mathcal{ND}^0$; Gentzen [**Gentzen:uudlsi35**])

- Idea: $\mathcal{ND}^0$ tries to mimic human theorem proving behavior (non-minimal)
- **Definition 4.1.** The propositional natural deduction calculus $\mathcal{ND}^0$ has rules for the introduction and elimination of connectives

Introduction	Elimination	Axiom
$\dfrac{\mathbf{A}\quad\mathbf{B}}{\mathbf{A}\wedge\mathbf{B}}\wedge I$	$\dfrac{\mathbf{A}\wedge\mathbf{B}}{\mathbf{A}}\wedge E_l$ $\quad \dfrac{\mathbf{A}\wedge\mathbf{B}}{\mathbf{B}}\wedge E_r$	$\dfrac{}{\mathbf{A}\vee\neg\mathbf{A}}\mathrm{TND}$
$\dfrac{\begin{array}{c}[\mathbf{A}]^1\\ \vdots \\ \mathbf{B}\end{array}}{\mathbf{A}\Rightarrow\mathbf{B}}\Rightarrow I^1$	$\dfrac{\mathbf{A}\Rightarrow\mathbf{B}\quad\mathbf{A}}{\mathbf{B}}\Rightarrow E$	

- TND is used only in classical logic (otherwise constructive/intuitionistic)

Recap Natural Deduction

More Rules for Natural Deduction

- **Definition 4.4.** $\mathcal{ND}^0$ has the following additional rules for the remaining connectives.

$$\frac{\mathbf{A}}{\mathbf{A} \vee \mathbf{B}} \vee I_l \qquad \frac{\mathbf{B}}{\mathbf{A} \vee \mathbf{B}} \vee I_r \qquad \frac{\mathbf{A} \vee \mathbf{B} \quad \begin{array}{c} [\mathbf{A}]^1 \\ \vdots \\ \mathbf{C} \end{array} \quad \begin{array}{c} [\mathbf{B}]^1 \\ \vdots \\ \mathbf{C} \end{array}}{\mathbf{C}} \vee E^1$$

$$\frac{\begin{array}{c} [\mathbf{A}]^1 \\ \vdots \\ F \end{array}}{\neg \mathbf{A}} \neg I^1 \qquad \frac{\neg \neg \mathbf{A}}{\mathbf{A}} \neg E$$

$$\frac{\neg \mathbf{A} \quad \mathbf{A}}{F} FI \qquad \frac{F}{\mathbf{A}} FE$$

Examples

Use the rules to proof a formula. This may require some creativity!
(see blackboard)

Problem 9.2

(have a look at the assignment)

Notation:
Stick to notations introduced in the lecture!

Problem 9.3.1

See Dennis' post in the forum!
(you can/should swap all `iff` Symbols with $\Leftrightarrow$)
In a general sense, you are asked to show

$$\mathit{Completeness} \Leftrightarrow \mathit{ModelExistence}$$

How could you do that? Maybe via showing

$$\mathit{Completeness} \Rightarrow \mathit{ModelExistence} \tag{1}$$

$$\mathit{ModelExistence} \Rightarrow \mathit{Completeness} \tag{2}$$

in (1) or (2) you may use $(\neg A \Leftrightarrow \neg B) \Leftrightarrow (A \Leftrightarrow B)$

Problem 9.3.2

here the same as above:

- you can/should swap all `iff` Symbols with $\Leftrightarrow$
- you may use $(\neg A \Leftrightarrow \neg B) \Leftrightarrow (A \Leftrightarrow B)$
- Hint: Proofs in a calculus are always of finite length
  try to come up with a way of using these finite proofs as finites subsets

Misc: Questions, Anecdotes & etc

Questions?